SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)	–	1(3)	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)	–	–	–	**2(2)**
4.	Determinants	1(1)*	1(2)*	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	–	1(2)	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	**3(9)**
7.	Integrals	2(2)#	1(2)*	1(3)*	–	**4(7)**
8.	Application of Integrals	–	1(2)	1(3)	–	**2(5)**
9.	Differential Equations	1(1)*	1(2)	1(3)	–	**3(6)**
10.	Vector Algebra	3(3)	1(2)*	–	–	**4(5)**
11.	Three Dimensional Geometry	2(2)#	1(2)	–	1(5)*	**4(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	2(2)# + 1(4)	1(2)	–	–	**4(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.
#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours *Maximum marks : 80*

General Instructions :

1. This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.
2. Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.
3. Both Part-A and Part-B have internal choices.

Part - A :

1. It consists of two Sections-I and II.
2. Section-I comprises of 16 very short answer type questions.
3. Section-II contains 2 case study-based questions.

Part - B :

1. It consists of three Sections-III, IV and V.
2. Section-III comprises of 10 questions of 2 marks each.
3. Section-IV comprises of 7 questions of 3 marks each.
4. Section-V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART - A

Section - I

1. Evaluate : $\int_0^{\pi/2} x\cos x\,dx$

OR

Evaluate : $\int \cos^3 x \sin x\,dx$

2. Check whether the function $f : N \to N$ defined by $f(x) = 4 - 3x$ is one-one or not.

3. Solve the differential equation $\dfrac{dy}{dx} = 2^{y-x}$.

OR

Solve the differential equation $\dfrac{dy}{dx} = \left(\dfrac{y}{x}\right)^{1/3}$.

4. Simplify : $\tan\theta\begin{bmatrix}\sec\theta & \tan\theta \\ \tan\theta & -\sec\theta\end{bmatrix} + \sec\theta\begin{bmatrix}-\tan\theta & -\sec\theta \\ -\sec\theta & \tan\theta\end{bmatrix}$

5. Find the direction cosines of the line that makes equal angles with the three axes in space.

OR

Find the vector equation of the symmetrical form of equation of straight line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$

6. Prove that the function $f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$ is one-one in the interval $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$.

7. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then find the probability of getting exactly one red ball.

OR

If $P(A)=\frac{2}{5}, P(B)=\frac{3}{10}$ and $P(A\cap B)=\frac{1}{5}$, then find the value of $P(A' \mid B')$.

8. Find the angle between the vectors $\vec{a}$ and $\vec{b}$ if $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=4\hat{i}+4\hat{j}-2\hat{k}$.

9. A matrix A of order 3×3 has determinant 5. What is the value of $|3A|$?

OR

If $f(x)=\begin{vmatrix}(1+x)^{17} & (1+x)^{19} & (1+x)^{23}\\ (1+x)^{23} & (1+x)^{29} & (1+x)^{34}\\ (1+x)^{41} & (1+x)^{43} & (1+x)^{47}\end{vmatrix} = A + Bx + Cx^2 + \ldots\ldots$, then prove that $A = 0$.

10. Find the vector in the direction of the vector $\hat{i}-2\hat{j}+2\hat{k}$ that has magnitude 9.

11. If E and F are events such that $0 < P(F) < 1$, then prove that $P(E \mid F)+P(\bar{E} \mid F)=1$

12. Find the direction cosines of the line joining $A(0, 7, 10)$ and $B(-1, 6, 6)$.

13. If $g(x) = x^2 - 4x - 5$, then prove that g is not one-one on R.

14. Find the projection of the vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$.

15. If a matrix has 12 elements, then it has ________ possible orders.

16. Evaluate : $\int 2^{2^{2^x}} 2^{2^x} 2^x \, dx$

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. The Government declare that farmers can get ₹ 200 per quintal for their potatoes on 1st February and after that, the price will be dropped by ₹ 2 per quintal per extra day. Ramu's father has 80 quintal of potatoes in the field and he estimates that crop is increasing at the rate of 1 quintal per day.

Based on the above information, answer the following question.

(i) If x is the number of days after 1st February, then price and quantity of potatoes respectively can be expressed as

(a) ₹ $(200 - 2x)$, $(80 + x)$ quintals

(b) ₹ $(200 - 2x)$, $(80 - x)$ quintals

(c) ₹ $(200 + x)$, 80 quintals

(d) None of these

(ii) Revenue R as a function of x can be represented as

(a) $R(x) = 2x^2 - 40x - 16000$ (b) $R(x) = -2x^2 + 40x + 16000$

(c) $R(x) = 2x^2 + 40x - 16000$ (d) $R(x) = 2x^2 - 40x - 15000$

(iii) Find the number of days after 1st February, when Ramu's father attain maximum revenue.

(a) 10 (b) 20 (c) 12 (d) 22

(iv) On which day should Ramu's father harvest the potatoes to maximise his revenue?

(a) 11th February (b) 20th Febraury (c) 12th February (d) 22nd February

(v) Maximum revenue is equal to

(a) ₹16000 (b) ₹18000 (c) ₹16200 (d) ₹16500

18. In an annual board examination, in a particular school, 30% of the students failed in Chemistry, 25% failed in Mathematics and 12% failed in both Chemistry and Mathematics. A student is selected at random.

(i) The probability that the selected student has failed in Chemistry, if it is known that he has failed in Mathematics, is

(a) $\frac{3}{10}$ (b) $\frac{12}{25}$

(c) $\frac{1}{4}$ (d) $\frac{3}{25}$

(ii) The probability that the selected student has failed in Mathematics, if it is known that he has failed in Chemistry, is

(a) $\frac{22}{25}$ (b) $\frac{12}{25}$ (c) $\frac{2}{5}$ (d) $\frac{3}{25}$

(iii) The probability that the selected student has passed in at least one of the two subjects, is

(a) $\frac{22}{25}$ (b) $\frac{88}{125}$ (c) $\frac{43}{100}$ (d) $\frac{3}{75}$

(iv) The probability that the selected student has failed in at least one of the two subjects, is

(a) $\frac{2}{5}$ (b) $\frac{22}{25}$ (c) $\frac{3}{5}$ (d) $\frac{43}{100}$

(v) The probability that the selected student has passed in Mathematics, if it is known that he has failed in Chemistry, is

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{4}{5}$

PART - B

Section - III

19. Find $\frac{dy}{dx}$ at $x = 1, y = \frac{\pi}{4}$, if $\sin^2 y + \cos xy = K$.

20. Find the value of $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$.

OR

Evaluate : $\int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x} dx$

21. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then find the probability that both drawn balls are black.

22. Find the number of solutions of the equation $2\cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6}$, if $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$.

23. Evaluate the determinant $\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$.

OR

If x is a complex root of the equation

$$\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix} + \begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 1-x \end{vmatrix} = 0,$$ then find the value of $x^{2007} + x^{-2007}$.

24. Find the solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$ satisfying $y(1) = 1$.

25. The x-coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z-coordinate.

26. Find the point on the curve $y = (x - 3)^2$ where the tangent is parallel to the chord joining (3, 0) and (4, 1).

27. Find the area bounded by the curve $y^2 = x$, line $y = 4$ and y-axis.

28. Find a unit vector perpendicular to the plane ABC, where A, B and C are the points (3, –1, 2), (1, –1, –3), (4, –3, 1) respectively.

OR

Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vectors. Show that the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

Section - IV

29. Show that the height of the closed cylinder of given surface area and maximum volume, is equal to the diameter of base.

30. Solve : $(x\sqrt{x^2 + y^2} - y^2)dx + xy\,dy = 0$

31. If $y = e^x \sin x^3 + (\tan x)^x$, then find $\frac{dy}{dx}$.

OR

If $x = 3 \sin t - \sin 3t$, $y = 3 \cos t - \cos 3t$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$.

32. Find the area bounded by the lines $y = 1 - ||x| - 1|$ and the x- axis.

33. Let $f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b, & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x, & \frac{\pi}{2} < x \le \pi \end{cases}$

be continuous in $[0, \pi]$, then find the value of $a + b$.

34. Evaluate : $\int_0^{\pi/2} \frac{\sin^2 x}{(1 + \sin x \cos x)} dx$

OR

Find the value of $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}\,dx$.

35. Show that the relation R in the set of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive, nor symmetric, nor transitive.

Section - V

36. Find the product BA of matrices $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it in solving the equations : $x + y + 2z = 1$; $3x + 2y + z = 7$; $2x + y + 3z = 2$.

OR

Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that $A\cdot(\text{adj } A) = |A|I_3$.

37. Solve the following linear programming problem graphically.

Minimize $Z = x - 7y + 227$

subject to constraints :

$x + y \leq 9$

$x \leq 7$

$y \leq 6$

$x + y \geq 5$

$x, y \geq 0$

OR

Solved the following linear programming problem graphically.

Maximize $Z = 11x + 9y$

subject to constraints :

$180x + 120y \leq 1500$

$x + y \leq 10$

$x, y \geq 0$

38. If the lines $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ and $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ are perpendicular, then find the value of k and hence find the equation of plane containing these lines.

OR

Find the equation of the plane that contains the point $(1, -1, 2)$ and is perpendicular to both the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$. Hence find the distance of point $P(-2, 5, 5)$ from the plane obtained above.

1. $\int_0^{\pi/2} x\cos x\,dx = [x\sin x]_0^{\pi/2} - \int_0^{\pi/2} 1\cdot\sin x\,dx$

[Integrating by parts]

$$= \frac{\pi}{2} + [\cos x]_0^{\pi/2} = \frac{\pi}{2} - 1$$

OR

We have, $\int \cos^3 x \sin x\; dx$

Put $\cos x = t \Rightarrow \sin x\, dx = -dt$

$\therefore \int \cos^3 x \sin x\, dx = -\int t^3 dt = -\frac{t^4}{4} + C$

$$= -\frac{1}{4}\cos^4 x + C$$

2. We have, $f: N \to N, f(x) = 4 - 3x$

Let $f(x_1) = f(x_2) \Rightarrow 4 - 3x_1 = 4 - 3x_2 \Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

3. We have, $\frac{dy}{dx} = 2^{y-x} \Rightarrow \frac{dy}{2^y} = \frac{dx}{2^x}$

Integrating both sides, we get

$$\frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$$

$\Rightarrow -2^{-y} + 2^{-x} = C\log 2 = k(\text{say}) \Rightarrow 2^{-x} - 2^{-y} = k$

OR

We have, $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3} \Rightarrow y^{-1/3}\,dy = x^{-1/3}\,dx$

Integrating both sides, we get

$\frac{3}{2}y^{2/3} = \frac{3}{2}x^{2/3} + k \Rightarrow y^{2/3} - x^{2/3} = \frac{2}{3}k = c\ (\text{say})$

Hence, required solution is $y^{2/3} - x^{2/3} = c$

4. We have,

$$\tan\theta\begin{bmatrix}\sec\theta & \tan\theta\\ \tan\theta & -\sec\theta\end{bmatrix} + \sec\theta\begin{bmatrix}-\tan\theta & -\sec\theta\\ -\sec\theta & \tan\theta\end{bmatrix}$$

$$= \begin{bmatrix}\tan\theta\sec\theta & \tan^2\theta\\ \tan^2\theta & -\tan\theta\sec\theta\end{bmatrix} + \begin{bmatrix}-\tan\theta\sec\theta & -\sec^2\theta\\ -\sec^2\theta & \tan\theta\sec\theta\end{bmatrix}$$

$$= \begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}.$$

5. Since $l = m = n$ and $l^2 + m^2 + n^2 = 1$

$\Rightarrow l = m = n = \pm\frac{1}{\sqrt{3}}.$

OR

The vector form of $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ is

$$\vec{r} = (x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$$

$\therefore$ Required equation in vector form is

$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})$

6. We have, $f(x) = 2\left(\frac{\sqrt{3}}{2}\sin 2x - \frac{1}{2}\cos 2x\right) + 4$

$= 2\left(\cos\frac{\pi}{6}\sin 2x - \sin\frac{\pi}{6}\cos 2x\right) + 4$

$\Rightarrow f(x) = 2\sin\left(2x - \frac{\pi}{6}\right) + 4$

$\because$ $\sin x$ is one-one in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore -\frac{\pi}{2} \le 2x - \frac{\pi}{6} \le \frac{\pi}{2} \Rightarrow x \in \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$

7. Required probability $= P\{(RBB), (BRB), (BBR)\}$

$= P(RBB) + P(BRB) + P(BBR)$

$= \frac{5}{8}\times\frac{3}{7}\times\frac{2}{6} + \frac{3}{8}\times\frac{5}{7}\times\frac{2}{6} + \frac{3}{8}\times\frac{2}{7}\times\frac{5}{6} = 3\times\frac{5}{56} = \frac{15}{56}$

OR

Given, $P(A) = \frac{2}{5}, P(B) = \frac{3}{10}, P(A\cap B) = \frac{1}{5}$

Now, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$$= \frac{2}{5} + \frac{3}{10} - \frac{1}{5} = \frac{1}{2}$$

$\therefore P(A'\cap B') = P((A\cup B)') = 1 - P(A\cup B) = 1 - \frac{1}{2} = \frac{1}{2}$

Also, $P(B') = 1 - P(B) = 1 - \frac{3}{10} = \frac{7}{10}$

$\therefore P(A'|B') = \frac{P(A'\cap B')}{P(B')} = \frac{1/2}{7/10} = \frac{5}{7}$

8. We have, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} + 4\hat{j} - 2\hat{k}$

Now, $\vec{a}\cdot\vec{b} = (2\hat{i} - \hat{j} + 2\hat{k})\cdot(4\hat{i} + 4\hat{j} - 2\hat{k})$

$= 8 - 4 - 4 = 0.$

So, angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$.

9. Given, $|A| = 5$, order of A is 3×3.

$\therefore |3A| = 3^3|A| = 27\times 5 = 135.$

OR

Given that,

$$f(x)=\begin{vmatrix}(1+x)^{17} & (1+x)^{19} & (1+x)^{23}\\ (1+x)^{23} & (1+x)^{29} & (1+x)^{34}\\ (1+x)^{41} & (1+x)^{43} & (1+x)^{47}\end{vmatrix}$$

$= A + Bx + Cx^2 + ...$

On putting $x = 0$, we have $f(0)=\begin{vmatrix}1&1&1\\1&1&1\\1&1&1\end{vmatrix}=A$

$\Rightarrow A = 0$ ($\because R_1$ and R_2 are identical)

10. Let $\vec{a}=\hat{i}-2\hat{j}+2\hat{k}$

$|\vec{a}|=\sqrt{1+4+4}=\sqrt{9}=3$

$\therefore$ Required vector $=\dfrac{9(\hat{i}-2\hat{j}+2\hat{k})}{3}=3(\hat{i}-2\hat{j}+2\hat{k})$

11. $P(E\,|\,F)+P(\bar{E}\,|\,F)$

$=\dfrac{P(E\cap F)+P(\bar{E}\cap F)}{P(F)}=\dfrac{P((E\cup\bar{E})\cap F)}{P(F)}=\dfrac{P(F)}{P(F)}=1$

12. Direction ratios of AB are

$(-1-0, 6-7, 6-10)$ or $(-1, -1, -4)$

Also, $\sqrt{(-1)^2+(-1)^2+(-4)^2}=3\sqrt{2}$

$\therefore$ Direction cosines are $\left(-\dfrac{1}{3\sqrt{2}}, -\dfrac{1}{3\sqrt{2}}, \dfrac{-4}{3\sqrt{2}}\right)$

or $\left(\dfrac{1}{3\sqrt{2}}, \dfrac{1}{3\sqrt{2}}, \dfrac{4}{3\sqrt{2}}\right)$

13. Let $g(x_1) = g(x_2)$

$\Rightarrow x_1^2-4x_1-5=x_2^2-4x_2-5$

$\Rightarrow x_1^2-x_2^2=4(x_1-x_2)$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 - 4) = 0$

Either $x_1 = x_2$ or $x_1 + x_2 = 4$

Either $x_1 = x_2$ or $x_1 = 4 - x_2$

$\therefore$ There are two values of x_1, for which $g(x_1) = g(x_2)$.

$\therefore$ $g(x)$ is not one-one $\forall\ x \in R$

14. We have, $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$

$\therefore\ \vec{a}\cdot\vec{b}=(2\hat{i}+3\hat{j}+2\hat{k})\cdot(\hat{i}+2\hat{j}+\hat{k})=2+6+2=10$

and $|\vec{b}|=\sqrt{1^2+2^2+1^2}=\sqrt{6}$

So, projection of $\vec{a}$ on $\vec{b}=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=\dfrac{10}{\sqrt{6}}$

15. All possible orders of 12 elements are

$1 \times 12, 12 \times 1, 2 \times 6, 6 \times 2, 3 \times 4, 4 \times 3$ *i.e.*, 6

16. Let $I=\int 2^{2^{2^x}}\, 2^{2^x}\, 2^x\, dx$

Put $2^{2^{2^x}}=t \Rightarrow 2^{2^{2^x}}\, 2^{2^x}\, 2^x(\log 2)^3\, dx=dt$

$\therefore\ I=\int\dfrac{1}{(\log 2)^3}\,dt=\dfrac{1}{(\log 2)^3}t+C=\dfrac{1}{(\log 2)^3}2^{2^{2^x}}+C$

17. (i) (a) : Let x be the number of extra days after 1st February

$\therefore$ Price = ₹$(200 - 2 \times x)$ = ₹$(200 - 2x)$

Quantity = 800 quintals + x(1 quintal per day)

= $(80 + x)$ quintals

(ii) (b) : $R(x)$ = Quantity × Price

$= (80 + x)(200 - 2x)$

$= 16000 - 160x + 200x - 2x^2$

$= 16000 + 40x - 2x^2$

(iii) (a) : We have, $R(x) = 16000 + 40x - 2x^2$

$\Rightarrow R'(x) = 40 - 4x \Rightarrow R''(x) = -4$

For $R(x)$ to be maximum, $R'(x) = 0$ and $R''(x) < 0$

$\Rightarrow 40 - 4x = 0 \Rightarrow x = 10$

(iv) (a) : Ramu's father will attain maximum revenue after 10 days.

So, he should harvest the potatoes after 10 days of 1st February *i.e.*, on 11th February.

(v) (c) : Maximum revenue is collected by Ramu's father when $x = 10$

$\therefore$ Maximum revenue $= R(10)$

$= 16000 + 40(10) - 2(10)^2$

$= 16000 + 400 - 200 = 16200.$

18. Let C denote the event that student has failed in Chemistry and M denote the event that student has failed in Mathematics.

$\therefore\ P(C)=\dfrac{30}{100}=\dfrac{3}{10},\ P(M)=\dfrac{25}{100}=\dfrac{1}{4}$

and $P(C\cap M)=\dfrac{12}{100}=\dfrac{3}{25}$

(i) (b) : Required probability = $P(C\,|\,M)$

$=\dfrac{P(C\cap M)}{P(M)}=\dfrac{3/25}{1/4}=\dfrac{12}{25}$

(ii) (c) : Required probability $P(M\,|\,C)$

$=\dfrac{P(M\cap C)}{P(C)}=\dfrac{3/25}{3/10}=\dfrac{2}{5}$

(iii) (a) : Revenue probability

$= P(C' \cup M') = P[C \cap M]'$

$=1-P(C\cap M)=1-\dfrac{3}{25}=\dfrac{22}{25}$

(iv) (d) : Required probability

$= P(C) + P(M) - P(C \cap M)$

$=\dfrac{3}{10}+\dfrac{1}{4}-\dfrac{3}{25}=\dfrac{43}{100}$

(v) (b) : Required probability = $P(M' \mid C)$

$$= \frac{P(M' \cap C)}{P(C)} = \frac{P(C) - P(C \cap M)}{P(C)} = \frac{9/50}{3/10} = \frac{3}{5}$$

19. We have, $\sin^2 y + \cos xy = K$

Differentiating w.r.t. x, we get

$$2\sin y \cos y \frac{dy}{dx} + (-\sin xy)\left(x\frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \sin xy}{\sin 2y - x\sin xy}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{\left(1, \frac{\pi}{4}\right)} = \frac{\frac{\pi}{4}\sin\frac{\pi}{4}}{\sin\frac{\pi}{2} - \sin\frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2}-1)}$$

20. Let $I = \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$

Putting $x = t^6 \Rightarrow dx = 6t^5\, dt$, we get

$$I = \int \frac{6t^5}{t^3 + t^2}\, dt = 6\int \frac{t^3}{t+1}\, dt = 6\int \left(t^2 - t + 1 - \frac{1}{t+1}\right) dt$$

$= 2t^3 - 3t^2 + 6t - 6\log(t+1) + C$

$= 2\sqrt{x} - 3(\sqrt[3]{x}) + 6(\sqrt[6]{x}) - 6\log(\sqrt[6]{x} + 1) + C$

OR

Let $I = \int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x}\, dx$

Dividing the numerator and denominator by $\cos^2 x$, we get

$$I = \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x + 8}\, dx$$

$$= \int \frac{\sec^2 x}{1 + \tan^2 x + 3\tan^2 x + 8}\, dx = \int \frac{\sec^2 x\, dx}{4\tan^2 x + 9}$$

Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$, we get

$$I = \int \frac{dt}{4t^2 + 9} = \frac{1}{4}\int \frac{dt}{t^2 + (3/2)^2} = \frac{1}{4} \times \frac{1}{3/2}\tan^{-1}\left(\frac{t}{3/2}\right) + C$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{2t}{3}\right) + C = \frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$$

21. Let E and F denote respectively the events that first and second ball drawn are black. We have to find $P(E \cap F)$ or $P(EF)$.

Now, $P(E) = P(\text{black ball in first draw}) = \frac{10}{15}$

When second ball is drawn without replacement, the probability that the second ball is black is the conditional probability of event F occurring when event E has already occurred.

$$\therefore \quad P(F \mid E) = \frac{9}{14}$$

By multiplication rule of probability, we have

$$P(E \cap F) = P(E) \,.\, P(F|E) = \frac{10}{15} \times \frac{9}{14} = \frac{3}{7}$$

22. Given equation is $2\cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$

$$\Rightarrow \cos^{-1} x + (\cos^{-1} x + \sin^{-1} x) = \frac{11\pi}{6}$$

$$\Rightarrow \cos^{-1} x + \frac{\pi}{2} = \frac{11\pi}{6}\left(\text{Given } \cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}\right)$$

$$\Rightarrow \cos^{-1} x = \frac{4\pi}{3},$$

which is not possible as $\cos^{-1} x \in [0, \pi]$.

Thus, given equation has no solution.

23. We have,

$$\Delta = \begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \Rightarrow \Delta = \begin{vmatrix} \log_3 2^9 & \log_{2^2} 3 \\ \log_3 2^3 & \log_{2^2} 3^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 9\log_3 2 & \frac{1}{2}\log_2 3 \\ 3\log_3 2 & \frac{2}{2}\log_2 3 \end{vmatrix} \quad \left[\because \log_{a^p} m^n = \frac{n}{p}\log_a m\right]$$

$$\Rightarrow \Delta = (9\log_3 2) \times (\log_2 3) - \left(\frac{1}{2}\log_2 3\right)(3\log_3 2)$$

$$\Rightarrow \Delta = 9(\log_3 2 \times \log_2 3) - \frac{3}{2}(\log_2 3 \times \log_3 2)$$

$$\Rightarrow \Delta = 9 - \frac{3}{2} \qquad [\because \log_b a \times \log_a b = 1]$$

$$\Rightarrow \Delta = \frac{15}{2}$$

OR

Expanding the two determinants, we get

$[1(1 - x^2) - x(x - x^2) + x(x^2 - x)] + [(1 - x)[(1 - x)^2 - 1) - 1(1 - x - 1) + 1(1 - 1 + x)]$

$\Rightarrow (1 - 3x^2 + 2x^3) + (3x^2 - x^3) = 0$

$\Rightarrow x^3 + 1 = 0 \Rightarrow x = -\omega, -\omega^2, -1$

$\therefore x^{2007} + x^{-2007} = -1 - 1 = -2.$

24. Given, $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$

$\Rightarrow 2xydy = (x^2 + y^2 + 1)dx \Rightarrow 2xydy - y^2dx = (x^2 + 1)dx$

$\Rightarrow xd(y^2) - y^2dx = (x^2 + 1)dx$

$$\Rightarrow \frac{xd(y^2) - y^2dx}{x^2} = \left(1 + \frac{1}{x^2}\right)dx \Rightarrow d\left(\frac{y^2}{x}\right) = d\left(x - \frac{1}{x}\right)$$

Integrating both sides, we get

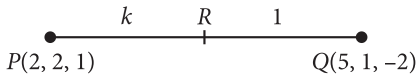
$$\frac{y^2}{x} = x - \frac{1}{x} + C \Rightarrow y^2 = x^2 - 1 + Cx$$

Now, given that $y(1) = 1$

$\therefore 1 = 1 - 1 + C \Rightarrow C = 1$

Thus, curve becomes $y^2 = x^2 - 1 + x$

25. Given that $P(2, 2, 1)$ and $Q(5, 1, -2)$



Let the point R on the line PQ, divides the line in the ratio $k : 1$. And x-coordinate of point R on the line is 4. So, by section formula

$$4 = \frac{5k+2}{k+1} \Rightarrow k = 2$$

Now, z-coordinate of point R,

$$z = \frac{-2k+1}{k+1} = \frac{-2\times 2+1}{2+1} = -1$$

$\Rightarrow$ z-coordinate of point $R = -1$

26. We have, $y = (x-3)^2$

Slope of tangent $= \frac{dy}{dx} = 2(x-3)$

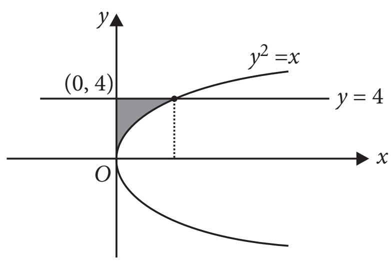
Slope of chord joining (3, 0) and (4, 1) $= \frac{1-0}{4-3} = 1$

For parallel lines, slopes are equal

$\therefore\ 2(x-3) = 1 \Rightarrow x = \frac{7}{2}$ and $y = \left(\frac{7}{2}-3\right)^2 = \frac{1}{4}$

Hence, required point is $\left(\frac{7}{2}, \frac{1}{4}\right)$.

27. We have, $y^2 = x$, a parabola with vertex (0, 0) and line $y = 4$



Required area = area of shaded region

$$= \int_0^4 y^2\,dy = \left[\frac{y^3}{3}\right]_0^4 = \frac{64}{3} \text{ sq. units}$$

28. The vector $\overrightarrow{AB}\times\overrightarrow{AC}$ is perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$\therefore$ Required vector $= \frac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC}|}$

Now, $\overrightarrow{AB}$ = P.V. of B – P.V. of A

$$= (\hat{i}-\hat{j}-3\hat{k})-(3\hat{i}-\hat{j}+2\hat{k}) = -2\hat{i}+0\hat{j}-5\hat{k}$$

and $\overrightarrow{AC}$ = P.V. of C – P.V. of A

$$= (4\hat{i}-3\hat{j}+\hat{k})-(3\hat{i}-\hat{j}+2\hat{k}) = \hat{i}-2\hat{j}-\hat{k}$$

$$\therefore\ \overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= (0-10)\hat{i}-(2+5)\hat{j}+(4-0)\hat{k} = -10\hat{i}-7\hat{j}+4\hat{k}$$

$$\Rightarrow |\overrightarrow{AB}\times\overrightarrow{AC}| = \sqrt{(-10)^2+(-7)^2+4^2} = \sqrt{165}$$

Hence, required vector

$$= \frac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC}|} = \frac{1}{\sqrt{165}}(-10\hat{i}-7\hat{j}+4\hat{k})$$

OR

Given, $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 3\hat{i}-\hat{j}+2\hat{k}$

Now, $\vec{a}+\vec{b} = 4\hat{i}+\hat{j}-\hat{k}$

Also, $\vec{a}-\vec{b} = -2\hat{i}+3\hat{j}-5\hat{k}$

Now, $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = (4\hat{i}+\hat{j}-\hat{k})\cdot(-2\hat{i}+3\hat{j}-5\hat{k})$

$= (4)(-2) + (1)(3) + (-1)(-5) = -8 + 3 + 5 = 0$

Hence, $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

29. Let r be the radius of the base and h be the height of a closed cylinder of given surface area S. Then,

$$S = 2\pi r^2 + 2\pi rh \Rightarrow h = \frac{S-2\pi r^2}{2\pi r} \quad ...(i)$$

Let V be the volume of the cylinder. Then,

$V = \pi r^2 h$

$$\Rightarrow V = \pi r^2\left(\frac{S-2\pi r^2}{2\pi r}\right) = \left(\frac{rS-2\pi r^3}{2}\right)$$

$$\Rightarrow \frac{dV}{dr} = \frac{S}{2} - 3\pi r^2$$

For maximum or minimum value of V, we have

$$\frac{dV}{dr} = 0 \Rightarrow \frac{S}{2} - 3\pi r^2 = 0 \Rightarrow S = 6\pi r^2$$

From(i), $h = \frac{6\pi r^2 - 2\pi r^2}{2\pi r} \Rightarrow h = 2r$

Also, $\frac{d^2V}{dr^2} = -6\pi r < 0$.

Hence, V is maximum when $h = 2r$ *i.e.*, when the height of the cylinder is equal to the diameter of the base.

30. The given equation can be written as

$$\frac{dy}{dx} = \frac{y^2 - x\sqrt{x^2+y^2}}{xy}, \text{ which is clearly homogeneous.}$$

Putting $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x\sqrt{x^2 + v^2x^2}}{vx^2}$$

$$\Rightarrow x\frac{dv}{dx} = \left(\frac{v^2 - \sqrt{1+v^2}}{v} - v\right) \Rightarrow x\frac{dv}{dx} = \frac{-\sqrt{1+v^2}}{v}$$

$$\Rightarrow \int \frac{v}{\sqrt{1+v^2}}dv = -\int \frac{dx}{x} \Rightarrow \sqrt{1+v^2} = -\log|x| + C$$

$$\Rightarrow \sqrt{x^2+y^2} + x\log|x| = Cx$$

31. Let $u = e^x \sin x^3$ and $v = (\tan x)^x$

Now, $u = e^x \sin x^3$

Differentiating w.r.t. x, we get

$$\frac{du}{dx} = e^x \cdot \frac{d\{\sin(x)^3\}}{dx} + \sin x^3 \cdot \frac{d}{dx}(e^x)$$

$$= e^x \cdot \cos x^3 \cdot 3x^2 + \sin x^3 \cdot e^x$$

Hence, $\frac{du}{dx} = 3x^2 \cdot e^x \cos x^3 + e^x \sin x^3$

Again, $v = (\tan x)^x \Rightarrow \log v = x \log(\tan x)$

Differentiating w.r.t. x, we get

$$\frac{1}{v}\frac{dv}{dx} = 1 \cdot \log(\tan x) + x \cdot \frac{1}{\tan x}\sec^2 x$$

$$\therefore \frac{dv}{dx} = v[\log(\tan x) + x \cot x \cdot \sec^2 x]$$

$$= (\tan x)^x [\log(\tan x) + x \cot x \sec^2 x]$$

Now, $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$$\Rightarrow \frac{dy}{dx} = 3x^2e^x\cos(x^3) + e^x\sin(x^3) + (\tan x)^x[\log(\tan x) + x\cot x \sec^2 x]$$

OR

We have, $x = 3\sin t - \sin 3t$

$$\Rightarrow \frac{dx}{dt} = 3\cos t - 3\cos 3t \quad \text{...(i)}$$

$$y = 3\cos t - \cos 3t \Rightarrow \frac{dy}{dt} = -3\sin t + 3\sin 3t \quad \text{...(ii)}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin 3t - \sin t}{\cos t - \cos 3t} \quad \text{[Dividing (ii) by (i)]}$$

$$= \frac{2\cos 2t \sin t}{2\sin 2t \sin t} = \cot 2t$$

Differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -2\operatorname{cosec}^2 2t \cdot \frac{dt}{dx}$$

$$= -2\operatorname{cosec}^2 2t \cdot \frac{1}{3(\cos t - \cos 3t)} \quad \text{[From (i)]}$$

At $t = \frac{\pi}{3}$,

$$\frac{d^2y}{dx^2} = -2\operatorname{cosec}^2\frac{2\pi}{3} \cdot \frac{1}{3\left(\cos\frac{\pi}{3} - \cos\frac{3\pi}{3}\right)}$$

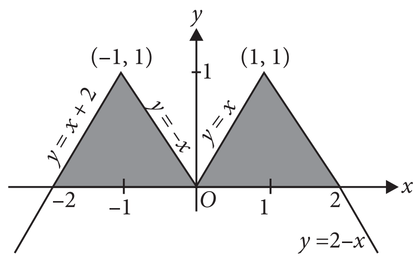
$$= -2\left(\frac{2}{\sqrt{3}}\right)^2 \cdot \frac{1}{3\left(\frac{1}{2}+1\right)} = -\frac{16}{27}$$

32. We have, $y = 1 - |x - 1|$ if $x \ge 0$

$$= \begin{cases} 1-(x-1), & \text{if } x \ge 1 \\ 1+(x-1), & \text{if } x < 1 \end{cases} = \begin{cases} 2-x, & \text{if } x \ge 1 \\ x, & \text{if } x < 1 \end{cases}$$

and $y = 1 - |-x - 1| = 1 - |x + 1|$, if $x < 0$

$$= \begin{cases} 1-(x+1), & \text{if } x \ge -1 \\ 1+(x+1), & \text{if } x < -1 \end{cases} = \begin{cases} -x, & \text{if } x \ge -1 \\ x+2, & \text{if } x < -1 \end{cases}$$



$$\text{Required area} = 2\left[\int_0^1 x\,dx + \int_1^2 (2-x)\,dx\right]$$

$$= 2\left[\frac{x^2}{2}\right]_0^1 + 2\left[2x - \frac{x^2}{2}\right]_1^2 = 1 + 1 = 2 \text{ sq. units}$$

33. Since, $f(x)$ is continuous at $x = \pi/4$.

$$\therefore \text{ L.H.L.}\left(\text{at } x = \frac{\pi}{4}\right) = f\left(\frac{\pi}{4}\right) = \text{R.H.L.}\left(\text{at } x = \frac{\pi}{4}\right)$$

$$\lim_{x \to \frac{\pi}{4}} (x + a\sqrt{2}\sin x) = 2 \times \frac{\pi}{4}\cot\frac{\pi}{4} + b$$

$$\Rightarrow \frac{\pi}{4} + a = \frac{\pi}{2} + b \Rightarrow a - b = \frac{\pi}{4} \quad \text{...(i)}$$

Also, $f(x)$ is continuous at $x = \frac{\pi}{2}$.

$$\therefore \text{ L.H.L.}\left(\text{at } x = \frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) = \text{R.H.L.}\left(\text{at } x = \frac{\pi}{2}\right)$$

$$\lim_{x \to \frac{\pi}{2}} (2 \times x\cot x + b) = \lim_{x \to \frac{\pi}{2}} (a\cos 2x - b\sin x)$$

$$\Rightarrow 2 \times \frac{\pi}{2}\cot\frac{\pi}{2} + b = a\cos 2 \times \frac{\pi}{2} - b\sin\frac{\pi}{2}$$

$$\Rightarrow b = -a - b \Rightarrow 2b = -a \quad \text{...(ii)}$$

From (i) and (ii), we get $a + b = \frac{\pi}{12}$.

34. Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{(1+\sin x \cos x)} dx$ ...(i)

Then, $I = \int_0^{\pi/2} \frac{\sin^2[(\pi/2)-x]}{1+\sin[(\pi/2)-x]\cos[(\pi/2)-x]} dx$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos^2 x}{(1+\sin x \cos x)} dx$ ...(ii)

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{(\sin^2 x + \cos^2 x)}{(1+\sin x \cos x)} dx = \int_0^{\pi/2} \frac{dx}{(1+\sin x \cos x)}$$

$$= \int_0^{\pi/2} \frac{\sec^2 x}{(\sec^2 x + \tan x)} dx$$

(By dividing numerator and denominator by $\cos^2 x$)

$$= \int_0^{\pi/2} \frac{\sec^2 x}{(1+\tan^2 x + \tan x)} dx$$

Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$,

When $x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t \to \infty$

$$\therefore \quad I = \int_0^{\infty} \frac{dt}{(t^2+t+1)} = \int_0^{\infty} \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \left[\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^{\infty}$$

$$= \frac{2}{\sqrt{3}}\left[\tan^{-1}(\infty) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right] = \frac{2}{\sqrt{3}} \cdot \left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \frac{2\pi}{3\sqrt{3}}$$

OR

Let $I = \int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x} dx$ ...(i)

$\Rightarrow \quad I = \int_{\pi/4}^{3\pi/4} \frac{\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}{1+\sin\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)} dx$ (By property)

$\Rightarrow \quad I = \int_{\pi/4}^{3\pi/4} \frac{(\pi - x)dx}{1+\sin x}$ ...(ii)

Adding (i) and (ii), we get

$$2I = \pi\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\sin x} = \pi\int_{\pi/4}^{3\pi/4} \frac{(1-\sin x)}{\cos^2 x} dx$$

$$= \pi\int_{\pi/4}^{3\pi/4} (\sec^2 x - \sec x \tan x)\, dx$$

$$= \pi[\tan x - \sec x]_{\pi/4}^{3\pi/4} = \pi\{(-1+\sqrt{2})-(1-\sqrt{2})\}$$

$$\Rightarrow \quad 2I = \pi(2\sqrt{2}-2) \quad \Rightarrow \quad I = \pi(\sqrt{2}-1)$$

35. Given relation is $R = \{(a, b): a \le b^2\}$

Reflexivity: Let $a \in$ real numbers.

$aRa \Rightarrow a \le a^2$

but if $a < 1$, then $a \not\le a^2$

For example, $a = \frac{1}{2} \Rightarrow a^2 = \frac{1}{4}$ so, $\frac{1}{2} \not\le \frac{1}{4}$

Hence, R is not reflexive.

Symmetricity : $aRb \Rightarrow a \le b^2$

But then $b \le a^2$ is not true

$\therefore \quad aRb \not\Rightarrow bRa$

For example, $a = 2, b = 5$

then $2 \le 5^2$ but $5 \le 2^2$ is not true.

Hence, R is not symmetric.

Transitivity : Let $a, b, c \in$ real numbers

Considering aRb and bRc

$aRb \Rightarrow a \le b^2$ and $bRc \Rightarrow b \le c^2$

$\Rightarrow a \le c^4 \not\Rightarrow aRc$

For example, if $a = 2, b = -3, c = 1$

$aRb \Rightarrow 2 \le 9$

$bRc \Rightarrow -3 \le 1$

$aRc \Rightarrow 2 \le 1$ is not true.

Hence, R is not transitive

36. We have, $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

So, $BA = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$\Rightarrow \quad BA = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \quad BA = 4I$

$\Rightarrow \quad B^{-1}(BA) = 4B^{-1}I$ [Pre multiplying by B^{-1}]

$\Rightarrow \quad 4B^{-1} = IA \quad \Rightarrow \quad B^{-1} = \frac{1}{4}A$ ...(i)

Now, given system of equations can be written as $BX = C$, where

$B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

or $X = B^{-1}C \Rightarrow X = \frac{1}{4}AC$ [Using (i)]

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$\therefore$ $x = 2, y = 1, z = -1.$

OR

We have, $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$$\Rightarrow |A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix}$$

$= -1(1-4) - (-2)(2+4) - 2(-4-2)$

$= 3 + 12 + 12 = 27$

Now, $A_{11} = -3, A_{12} = -6, A_{13} = -6,$

$A_{21} = 6, A_{22} = 3, A_{23} = -6,$

$A_{31} = 6, A_{32} = -6, A_{33} = 3$

$$\therefore \text{ adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$\text{Now, } A\cdot(\text{adj } A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\cdot\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix} = 27\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A|I_3.$$

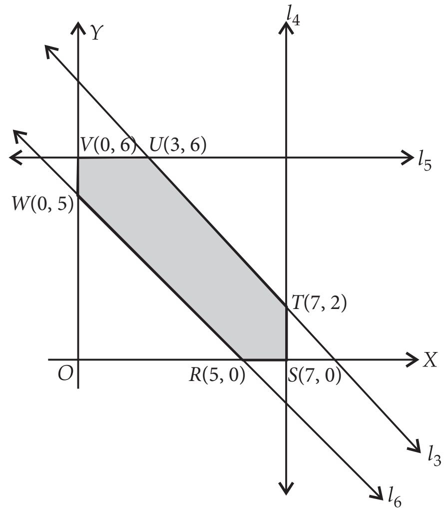
37. We have

Minimize $Z = x - 7y + 227$, subject to the constraints $x \geq 0, y \geq 0, x + y \leq 9, x \leq 7, y \leq 6$ and $x + y \geq 5$.

We draw the graphs of the lines

$l_1 : x = 0$, $l_2 : y = 0$, $l_3 : x + y = 9$, $l_4 : x = 7$, $l_5 : y = 6$ and $l_6 : x + y = 5$

as shown below.



Now, the intersection point of l_3 and l_4 is (7, 2).

Similarly, the intersection point of l_3 and l_5 is (3, 6).

Thus, the shaded region represents the feasible region whose vertices are R, S, T, U, V and W.

Corner points	Value of $Z = x - 7y + 227$
R (5, 0)	$5 - 0 + 227 = 232$
S (7, 0)	$7 - 0 + 227 = 234$
T (7, 2)	$7 - 7 \times 2 + 227 = 220$
U (3, 6)	$3 - 7 \times 6 + 227 = 188$
V (0, 6)	$0 - 7 \times 6 + 227 = 185$ (Minimum)
W (0, 5)	$0 - 7 \times 5 + 227 = 192$

Thus, Z is minimum at $x = 0$ and $y = 6$ and minimum value of z is 185.

OR

We have,

Maximize $Z = 11x + 9y$

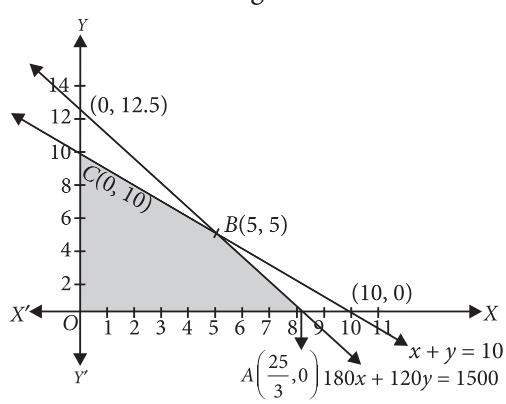
Subject to the constraints,

$180x + 120y \leq 1500$ ... (i)

$x + y \leq 10$ ... (ii)

$x, y \geq 0$ ... (iii)

Now, plotting the graph of (i), (ii) and (iii), we get the required feasible region (shaded) as shown below. We observe that the feasible region is bounded.



We have corner points as,

$A\left(\frac{25}{3},0\right), B(5,5)$ and $C(0,10)$.

Corner points	Value of $Z = 11x + 9y$
$A\left(\frac{25}{3},0\right)$	$\left(11\times\frac{25}{3}\right)+(9\times 0)=91.67$
$B(5, 5)$	$(11 \times 5) + (9 \times 5) = 100$ (Maximum)
C(0, 10)	$(11 \times 0) + (9 \times 10) = 90$

Thus, Z is maximum at $x = 5$ and $y = 5$ and maximum value of Z is 100.

38. The given lines are

$$\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2} \quad ...(i)$$

and $$\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5} \quad ...(ii)$$

Since, both the lines are perpendicular.

$\therefore\ -3\cdot k - 2k\cdot 1 + 2\cdot 5 = 0$

$\Rightarrow -5k + 10 = 0 \Rightarrow k = 2.$

Now equation of plane containing lines (i) & (ii) is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$

$\Rightarrow (x-1)(-20-2)-(y-2)(-15-4)+(z-3)(-3+8)=0$

$\Rightarrow -22(x-1)+19(y-2)+5(z-3)=0$

$\Rightarrow -22x + 19y + 5z = 31.$

OR

Given planes are $2x + 3y - 2z = 5$ ...(i)

and $x + 2y - 3z = 8$ ...(ii)

Normal vectors of (i) and (ii) are respectively

$\vec{m}_1 = 2\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{m}_2 = \hat{i}+2\hat{j}-3\hat{k}$

Since required plane is perpendicular to the planes (i) and (ii). So, normal to the required plane will be in the direction of $\vec{m}_1 \times \vec{m}_2$.

$$\text{So, } \vec{m}_1 \times \vec{m}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -2 \\ 1 & 2 & -3 \end{vmatrix} = -5\hat{i}+4\hat{j}+\hat{k}=\vec{m}.$$

Also position vector of point $(1, -1, 2)$ on the plane is, $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$.

So, equation of plane is $\vec{r}\cdot\vec{m}=\vec{a}\cdot\vec{m}$

$\Rightarrow \vec{r}\cdot(-5\hat{i}+4\hat{j}+\hat{k})=(\hat{i}-\hat{j}+2\hat{k})\cdot(-5\hat{i}+4\hat{j}+\hat{k})$

$\Rightarrow 5x - 4y - z - 7 = 0$ ...(iii)

Distance of $P(-2, 5, 5)$ from (iii) is,

$$d=\left|\frac{5(-2)-4(5)-5-7}{\sqrt{5^2+(-4)^2+(-1)^2}}\right|=\frac{42}{\sqrt{42}}=\sqrt{42}\text{ units}.$$

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